Tutorial 2

The Derivative

Outline

- The Slope of a Straight Line
- The Slope of a Curve at a Point
- The Derivative
- Limits and the Derivative
- Some Rules for Differentiation
- More About Derivatives
- The Derivative as a Rate of Change

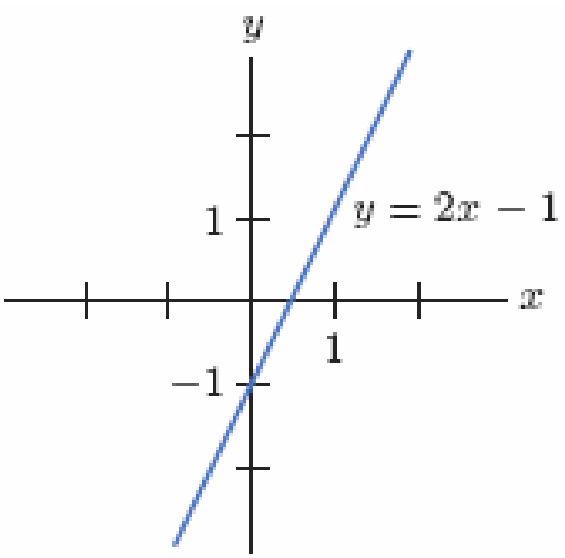
Nonvertical Lines

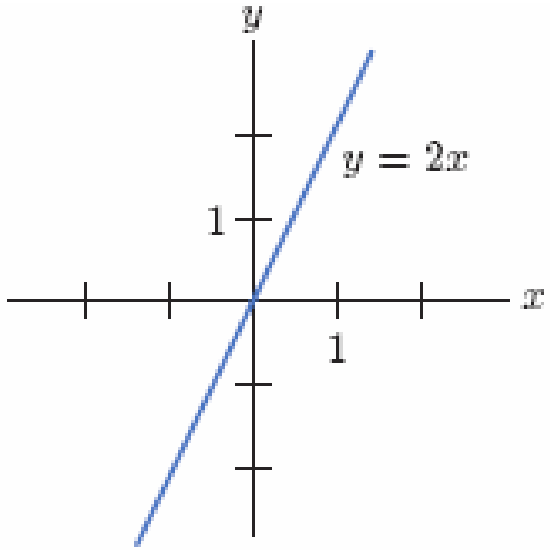
Definition	Example
Equations of Non-vertical Lines: A non-vertical line L has an equation of the form $$y = mx + b$$ The number m is called the slope of L and the point $(0, b)$ is called the y-intercept. The equation above is called the *slope-intercept equation* of L.	$$y = 3x - 4$$ For this line, $m = 3$ and $b = -4$.

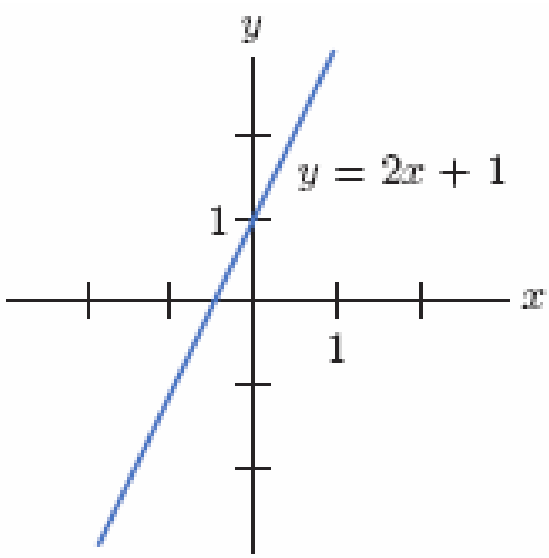
Lines – Positive Slope

EXAMPLE

The following are graphs of equations of lines that have positive slopes:



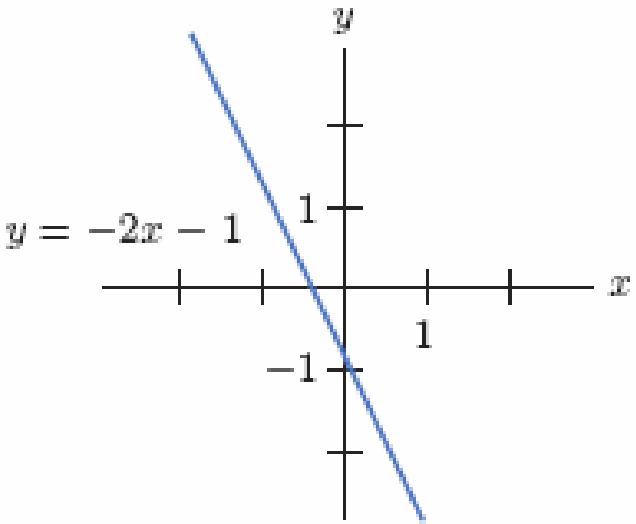


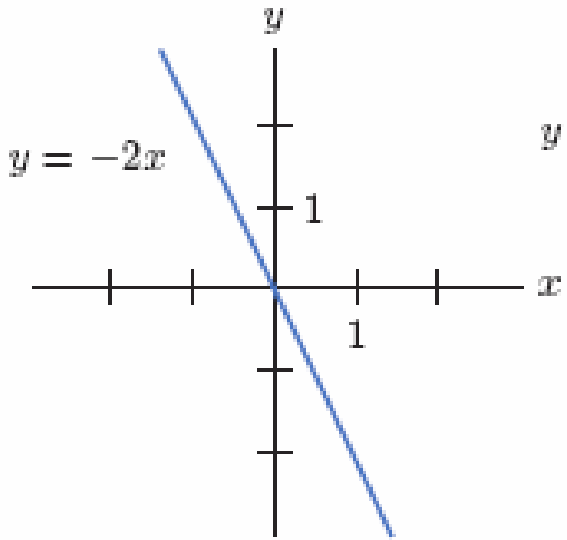


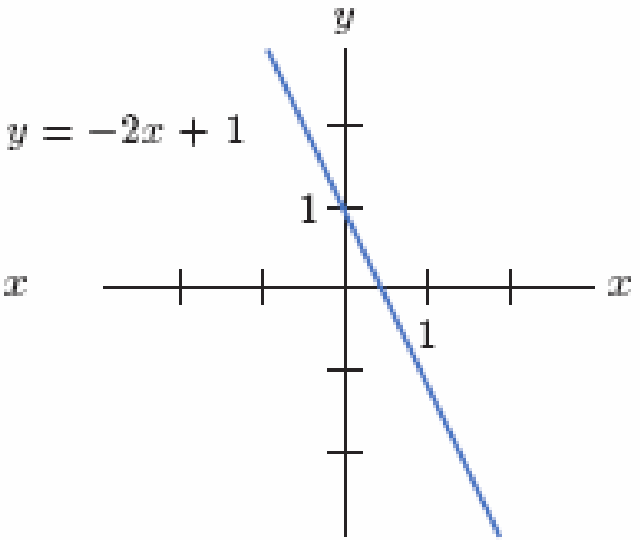
Lines – Negative Slope

EXAMPLE

The following are graphs of equations of lines that have negative slopes:







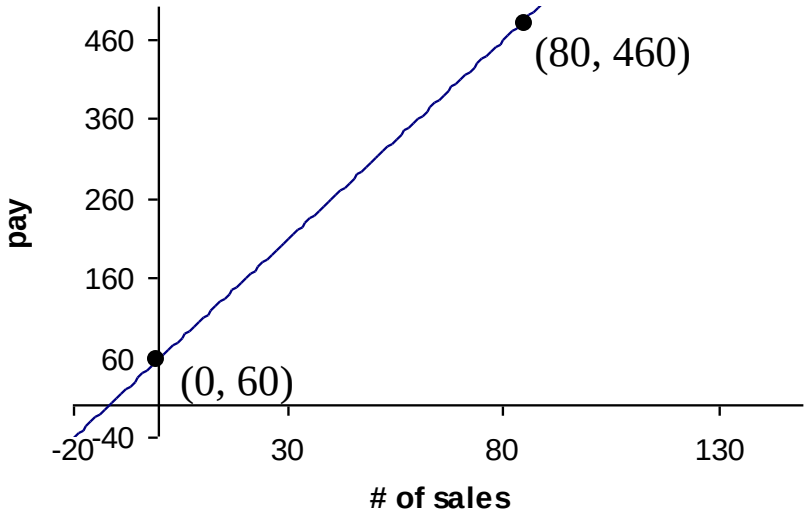
Interpretation of a Graph

EXAMPLE

A salesperson's weekly pay depends on the volume of sales. If she sells x units of goods, then her pay is $y = 5x + 60$ dollars. Give an interpretation of the slope and the y-intercept of this straight line.

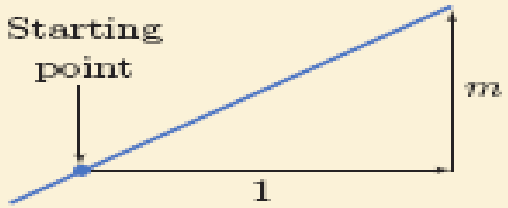
SOLUTION

The slope is 5, or 5/1. Since the numerator of this fraction represents the amount of change in her pay relative to the amount of change in her sales, the denominator, for every 1 sale that she makes, her pay increases by 5 dollars. The y-intercept is 60 and occurs on the graph at the point (0, 60). This point suggests that when she has executed 0 sales, her pay is 60 dollars. This \$60 could be referred to as her *base pay*.

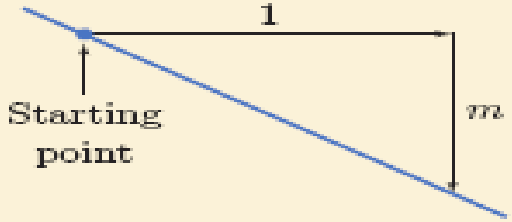


Properties of the Slope of a Nonvertical Line

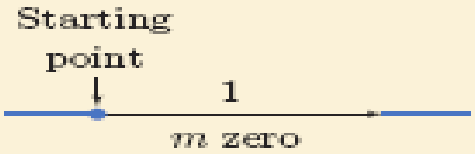
Slope Property 1 Suppose that we start at a point on a line of slope m and move one unit to the right. Then we must move m units in the y-direction in order to return to the line.



m positive

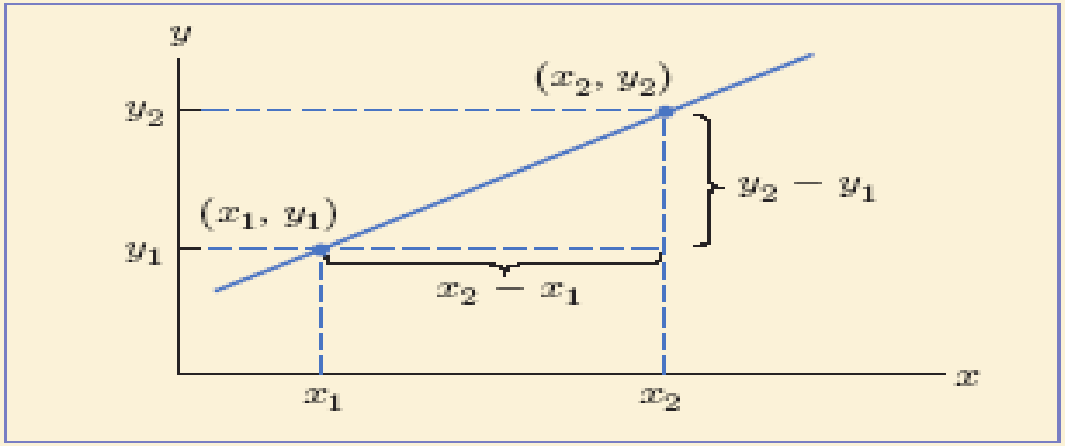


m negative



Slope Property 2 We can compute the slope of a line by knowing two points on the line. If (x_1, y_1) and (x_2, y_2) are on the line, then the slope of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$



Properties of the Slope of a Line

Slope Property 3 The equation of a line can be obtained if we know the slope and one point on the line. If the slope is m and if (x_1, y_1) is on the line, then the equation of the line is

$$y - y_1 = m(x - x_1). \tag{3}$$

This equation is called the *point-slope form* of the equation of the line.

Slope Property 4 Distinct lines of the same slope are parallel. Conversely, if two lines are parallel, they have the same slope.

Slope Property 5 When two lines are perpendicular, the product of their slopes is -1.

Finding Slope and y-intercept of a Line

EXAMPLE

Find the slope and y-intercept of the line $y = \frac{x+1}{3}$.

SOLUTION

First, we write the equation in *slope-intercept form*.

$y = \frac{x+1}{3}$	This is the given equation.
$y = \frac{x}{3} + \frac{1}{3}$	Divide both terms of the numerator of the right side by 3.
$y = \frac{1}{3}x + \frac{1}{3}$	Rewrite $\frac{x}{3}$ as $\frac{1}{3}x$.

Since the number being *multiplied* by x is 1/3, 1/3 is the slope of the line. Since the other 1/3 is the number being *added* to the term *containing* x, 1/3, or (0, 1/3), is the y-intercept.

Incidentally, it was a complete coincidence that the slope and y-intercept were the same number. This does not normally occur.

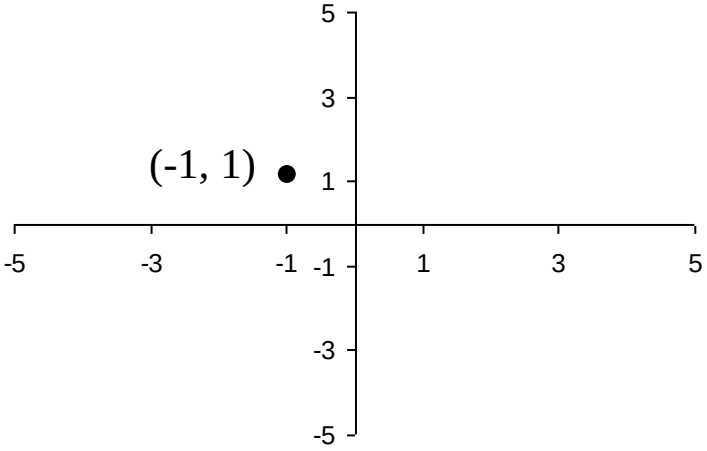
Sketching Graphs of Lines

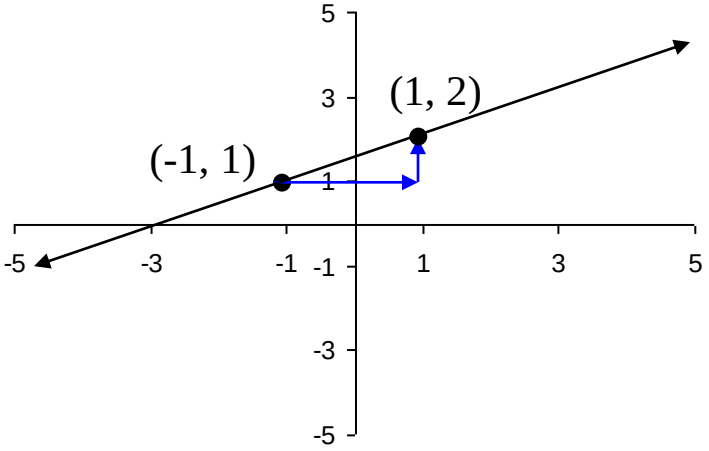
EXAMPLE

Sketch the graph of the line passing through (–1, 1) with slope ½.

SOLUTION

We use Slope Property 1. We begin at the given point (–1, 1) and from there, move up one unit and to the right two units to find another point on the line. We connect the two points that have already been determined, since two points determine a straight line.





Making Equations of Lines

EXAMPLE

Find an equation of the line that passes through the points (−1, 0) and (1, 2).

SOLUTION

Using the two points we will determine the slope by using Slope Property 2.

$$m = \frac{2-0}{1-(-1)} = \frac{2}{2} = 1$$

$y - y_1 = m(x - x_1)$	This is the equation from Property 3.
$y - 2 = 1(x - 1)$	$(x_1, y_1) = (1, 2)$ and $m = 1$.
$y - 2 = x - 1$	Distribute.
$y = x + 1$	Add 2 to both sides of the equation.

Making Equations of Lines

EXAMPLE

Find an equation of the line that passes through the point (2, 0) and is perpendicular to the line $y = 2x$.

SOLUTION

We know that the line we desire is perpendicular to the line $y = 2x$. Using Slope Property 5, we know that the product of the slope of the line desired and the slope of the line $y = 2x$ is -1. We recognize that the line $y = 2x$ is in slope-intercept form and therefore the slope of the line is 2. We can now find the slope of the line that we desire. Let the slope of the new line be m.

(*slope of a line*)(*slope of a new line*) $= -1$

$2m = -1$

$m = -0.5$

The slope of one line is 2 and the slope of the desired line is denoted by m.

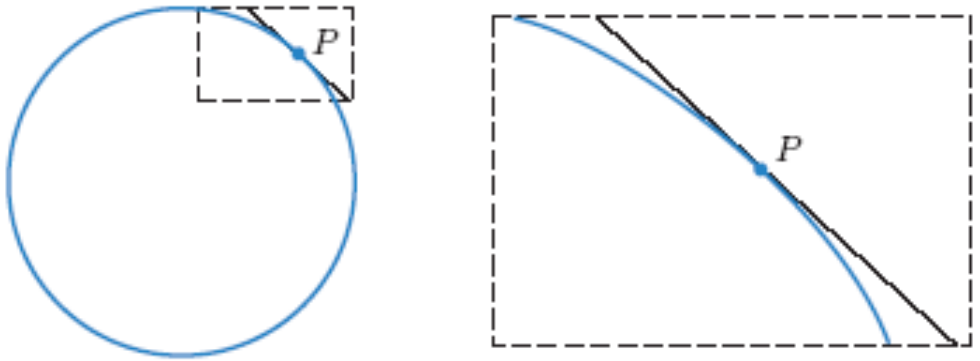
Now we can find the equation of the desired line using Property 3.

$y - y_1 = m(x - x_1)$ This is the equation from Property 3.

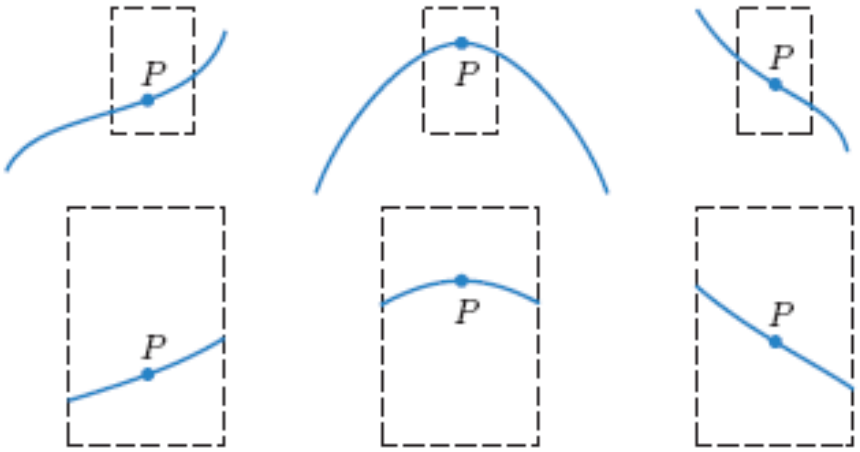
$y - 0 = -0.5(x - 2)$ $(x_1, y_1) = (2, 0)$ and $m = -0.5$.

$y = -0.5x + 1$ Distribute.

Tangent Lines

Definition	Example
Tangent Line to a Circle at a Point P: The straight line that touches the circle at just the one point *P*	

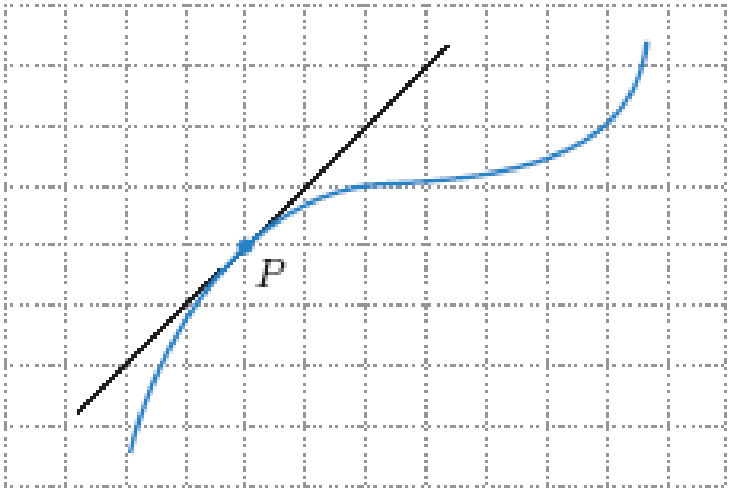
Slope of a Curve & Tangent Lines

Definition	Example
The Slope of a Curve at a Point P: The slope of the tangent line to the curve at *P*	 (Enlargements)

Slope of a Graph

EXAMPLE

Estimate the slope of the curve at the designated point P.



SOLUTION

The slope of a graph at a point is by definition the slope of the tangent line at that point. The figure above shows that the tangent line at P rises one unit for each unit change in x. Thus the slope of the tangent line at P is

$$\frac{\text{change in } y}{\text{change in } x} = \frac{1}{1} = 1.$$

Slope of a Curve: Rate of Change

Slope of a Curve as a Rate of Change The slope of a curve at a point P—that is, the slope of the tangent line at P—measures the rate of change (the rate of increase or decrease) of the curve as it passes through P.

The Derivative (Differentiation)

Definition	Example
Derivative: The slope formula for a function $y = f(x)$, denoted: $$y = f'(x)$$ *Differentiation*: The process of computing a derivative.	Given the function $f(x) = x^3$, the derivative is $$f'(x) = 3x^2$$

Differentiation: Rules

If $f(x) = mx + b$, then we have

$$f'(x) = m.$$

The derivative of a constant function $f(x) = b$ is zero; that is,

$$f'(x) = 0.$$

If $f(x) = x^2$, then its derivative is the function $2x$. That is,

$$f'(x) = 2x.$$

If $f(x) = x^3$, then the derivative is $3x^2$. That is,

$$f'(x) = 3x^2.$$

These examples can be summarized by the following rule:

Power Rule Let r be any number and let $f(x) = x^r$. Then $f'(x) = rx^{r-1}$.

Differentiation: Illustration

$f(x)$	$f'(x)$
$1 = x^0$	0
$x = x^1$	$1x^0 = 1$
x^2	$2x$
x^3	$3x^2$
x^4	$4x^3$

$f(x)$	7	x^3	x^{-2}	a	3×10^6	$1/x$
$f'(x)$	0	$3x^2$	$-2x^{-3}$	0	0	$-x^{-2}$

Differentiation: Examples

EXAMPLE

Find the derivative of $f(x) = \frac{1}{\sqrt[7]{x}}$.

SOLUTION

$$f(x) = \frac{1}{\sqrt[7]{x}}$$

This is the given equation.

$$f(x) = \frac{1}{x^{1/7}}$$

Rewrite the denominator as an exponent.

$$f(x) = x^{-1/7}$$

Rewrite with a negative exponent.

What we've done so far has been done for the sole purpose of rewriting the function in the form of $f(x) = x^r$.

$$f'(x) = -\frac{1}{7}x^{-1/7-1} = -\frac{1}{7}x^{-1/7-7/7} = -\frac{1}{7}x^{-8/7}$$

Use the Power Rule where $r = -1/7$ and then simplify.

Differentiation Examples

EXAMPLE

Find the slope of the curve $y = x^5$ at $x = -2$.

SOLUTION

We must first find the derivative of the given function.

$f(x) = x^5$ This is the given function.

$f'(x) = 5x^4$ Use the Power Rule.

Since the derivative function yields information about the slope of the original function, we can now use $f`(x)$ to determine the slope of the original function at $x = -2$.

$f'(-2) = 5(-2)^4$ Replace x with -2.

$f'(-2) = 5(16) = 80$ Evaluate.

Therefore, the slope of the original function at $x = -2$ is 80 (or 80/1).

Equation & Slope of a Tangent Line

Equation of the Tangent Line

$$y - f(a) = f'(a)(x - a)$$

EXAMPLE

Find the slope of the tangent line to the graph of $y = x^2$ at the point $(-0.4, 0.16)$ and then write the corresponding equation of the tangent line.

SOLUTION

The slope of the graph of $y = x^2$ at the point (x, y) is $2x$. The x-coordinate of $(-0.4, 0.16)$ is -0.4, so the slope of $y = x^2$ at this point is $2(-0.4) = -0.8$.

We shall write the equation of the tangent line in point-slope form. The point is $(-0.4, 0.16)$ and the slope (which we just found) is -0.8. Hence the equation is:

$$y - 0.16 = -0.8[x - (-0.4)]$$

$$y = -0.8x - 0.16$$

Equation of the Tangent Line

EXAMPLE

Find the equation of the tangent line to the graph of $f(x) = 3x$ at $x = 4$.

SOLUTION

We must first find the derivative of the given function.

$f(x) = 3x$ This is the given function.

$f'(x) = 3$ Differentiate.

Notice that in this case the derivative function is a constant function, 3. Therefore, at $x = 4$, or any other value, the value of the derivative will be 3. So now we use the Equation of the Tangent Line that we just saw.

$y - f(4) = f'(4)(x - 4)$ This is the Equation of the Tangent Line.

$y - 12 = 3(x - 4)$ $f(4) = 12$ and $f'(x) = 3$.

$y = 3x$ Simplify.

Leibniz Notation for Derivatives

Notation The operation of forming a derivative $f'(x)$ from a function $f(x)$ is also indicated by the symbol $\frac{d}{dx}$ (read "the derivative with respect to x"). Thus

$$\frac{d}{dx}f(x) = f'(x).$$

For example,

$$\frac{d}{dx}(x^6) = 6x^5, \qquad \frac{d}{dx}(x^{5/3}) = \frac{5}{3}x^{2/3}, \qquad \frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}.$$

When working with an equation of the form $y = f(x)$, we often write $\frac{dy}{dx}$ as a symbol for the derivative $f'(x)$. For example, if $y = x^6$, we may write

$$\frac{dy}{dx} = 6x^5.$$

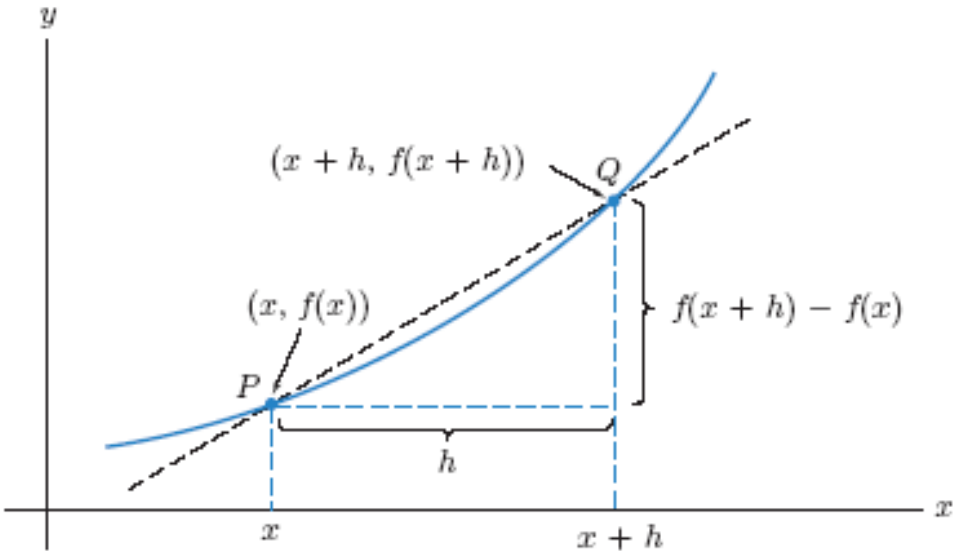
Ultimately, this notation is a better and more effective notation for working with derivatives.

Calculating Derivatives Via the Difference Quotient

The Difference Quotient is $\dfrac{f(x+h)-f(x)}{h}$.

To calculate $f'(x)$,

1. First calculate the quotient $\dfrac{f(x+h)-f(x)}{h}$ for $h \neq 0$.
2. Then let h approach zero.
3. The quantity $\dfrac{f(x+h)-f(x)}{h}$ will approach $f'(x)$.



Calculating Derivatives Via the Difference Quotient

EXAMPLE

Apply the three-step method to compute the derivative of the following function: $f'(x)$, where $f(x) = -x^2 + 2$.

SOLUTION

STEP 1: We calculate the difference quotient and simplify as much as possible.

$$\frac{f(x+h) - f(x)}{h} =$$

This is the difference quotient.

$$= \frac{\left[-(x+h)^2 + 2\right] - \left[-x^2 + 2\right]}{h}$$

Evaluate $f(x + h)$ and $f(x)$.

$$= \frac{\left[-\left(x^2 + 2xh + h^2\right) + 2\right] + x^2 - 2}{h}$$

Simplify.

$$= \frac{-x^2 - 2xh - h^2 + 2 + x^2 - 2}{h}$$

Simplify.

$$= \frac{-2xh - h^2}{h}$$

Simplify.

Calculating Derivatives Via the Difference Quotient

CONTINUED

$$= \frac{(-2x-h)\cancel{h}}{\cancel{h}}$$ Factor.

$$= -2x - h$$ Cancel and simplify.

STEP 2: As h approaches zero, the expression $-2x - h$ approaches $-2x$, and hence the difference quotient approaches $-2x$.

STEP 3: Since the difference quotient approaches the derivative of $f(x) = -x^2 + 2$ as h approaches zero, we conclude that $f'(x) = -2x$.

Using Limits to Calculate a Derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

EXAMPLE

Use the definition of the derivative to compute the derivative $f'(x)$ if $f(x) = \frac{1}{2x+5}$.

SOLUTION

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{2(x+h)+5} - \frac{1}{2x+5}}{h} = \lim_{h\to 0} \frac{\frac{2x+5}{(2x+2h+5)(2x+5)} - \frac{2x+2h+5}{(2x+2h+5)(2x+5)}}{h}$$

$$= \lim_{h\to 0} \frac{2x+5-2x-2h-5}{h(2x+2h+5)(2x+5)} = \lim_{h\to 0} \frac{-2h}{h(2x+2h+5)(2x+5)} = \lim_{h\to 0} \frac{-2}{(2x+2h+5)(2x+5)}$$

$$= \frac{-2}{(2x+5)(2x+5)} = -\frac{2}{(2x+5)^2}$$

Using Limits to Calculate a Derivative

EXAMPLE

Use the definition of the derivative to compute the derivative $f'(x)$ for the function $f(x)=\sqrt{x}$.

SOLUTION

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$=\lim_{h\to 0}\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\lim_{h\to 0}\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\lim_{h\to 0}\frac{1}{\left(\sqrt{x+h}+\sqrt{x}\right)}=\frac{1}{2\sqrt{x}}$$

Using Limits to Calculate a Derivative

Use the definition of the derivative to compute the derivative of the function

$$f(x) = 3x^3 + 2x - 1.$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\left[3(x+h)^3 + 2(x+h) - 1\right] - (3x^3 + 2x - 1)}{h}$$

$$= \lim_{h \to 0} \frac{3(x^3 + 3x^2h + 3xh^2 + h^3) + (2x + 2h) - 1 - 3x^3 - 2x + 1}{h}$$

$$= \lim_{h \to 0} \frac{9x^2h + 9xh^2 + 3h^3 + 2h}{h} = \lim_{h \to 0} (9x^2 + 9xh + 3h^2 + 2)$$

$$= 9x^2 + 0 + 0 + 2 = 9x^2 + 2$$

Rules of Differentiation

1. Constant-multiple rule

$$\frac{d}{dx}[k \cdot f(x)] = k \cdot \frac{d}{dx}[f(x)], \quad k \text{ a constant.}$$

2. Sum rule

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)].$$

3. General power rule

$$\frac{d}{dx}\left([g(x)]^r\right) = r \cdot [g(x)]^{r-1} \cdot \frac{d}{dx}[g(x)].$$

Differentiation

EXAMPLE

Differentiate $f(x) = \dfrac{45}{1 + x + \sqrt{x}}$.

SOLUTION

$$f(x) = \frac{45}{1 + x + \sqrt{x}}$$

This is the given function.

$$f'(x) = \frac{d}{dx}\left(\frac{45}{1 + x + \sqrt{x}}\right)$$

We begin to differentiate.

$$f'(x) = \frac{d}{dx}\left[45\left(1 + x + \sqrt{x}\right)^{-1}\right]$$

Rewrite the rational expression with a negative exponent.

$$f'(x) = 45\frac{d}{dx}\left[\left(1 + x + \sqrt{x}\right)^{-1}\right]$$

Use the Constant Multiple Rule.

$$f'(x) = 45(-1)\left(1 + x + \sqrt{x}\right)^{-2} \cdot \frac{d}{dx}\left(1 + x + \sqrt{x}\right)$$

Use the General Power Rule taking $1 + x + \sqrt{x}$ to be $g(x)$.

Differentiation

CONTINUED

$$f'(x) = 45(-1)\left(1 + x + \sqrt{x}\right)^{-2} \cdot \left(\frac{d}{dx}1 + \frac{d}{dx}x + \frac{d}{dx}x^{1/2}\right)$$

Use the Sum Rule and rewrite $\sqrt{x}$ as $x^{1/2}$.

$$f'(x) = 45(-1)\left(1 + x + \sqrt{x}\right)^{-2} \cdot \left(0 + 1 + \frac{1}{2}x^{-1/2}\right)$$

Differentiate.

$$f'(x) = 45(-1)\left(1 + x + \sqrt{x}\right)^{-2} \cdot \left(1 + \frac{1}{2}x^{-1/2}\right)$$

Simplify.

$$f'(x) = -45\frac{1}{\left(1 + x + \sqrt{x}\right)^2}\left(1 + 0.5\frac{1}{\sqrt{x}}\right)$$

Simplify.

$$f'(x) = -\frac{45\left(1 + \frac{0.5}{\sqrt{x}}\right)}{\left(1 + x + \sqrt{x}\right)^2}$$

Simplify.

Differentiation

EXAMPLE

Find the points on the graph of $f(x)=\dfrac{1}{x^3-12x+1}$ at which the tangent line is horizontal.

SOLUTION

$$f(x)=\frac{1}{x^3-12x+1}$$ This is the given function.

$$f'(x)=\frac{d}{dx}\left(\frac{1}{x^3-12x+1}\right)$$ We begin to differentiate.

$$f'(x)=\frac{d}{dx}\left[\left(x^3-12x+1\right)^{-1}\right]$$ Rewrite the rational expression with a negative exponent.

$$f'(x)=-1\left(x^3-12x+1\right)^{-2}\cdot\frac{d}{dx}\left(x^3-12x+1\right)$$ Use the General Power Rule taking $x^3-12x+1$ to be $g(x)$.

$$f'(x)=-1\left(x^3-12x+1\right)^{-2}\cdot\left(\frac{d}{dx}x^3-\frac{d}{dx}12x+\frac{d}{dx}1\right)$$ Use the Sum Rule.

Differentiation

CONTINUED

$$f'(x) = -1\left(x^3 - 12x + 1\right)^{-2} \cdot \left(3x^2 - 12 + 0\right)$$ Differentiate.

$$f'(x) = -1\left(x^3 - 12x + 1\right)^{-2} \cdot \left(3x^2 - 12\right)$$ Simplify.

$$f'(x) = -1\frac{1}{\left(x^3 - 12x + 1\right)^2}\left(3x^2 - 12\right)$$ Simplify.

$$f'(x) = -\frac{3x^2 - 12}{\left(x^3 - 12x + 1\right)^2}$$ Simplify.

$$f'(x) = -\frac{3x^2 - 12}{\left(x^3 - 12x + 1\right)^2} = 0$$ Tangent line is horizontal when the slope (derivative) is 0.

$$3x^2 - 12 = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$$ Solve the equation.

$$\left(2, -\frac{1}{15}\right), \left(-2, \frac{1}{17}\right)$$ Points at which the tangent line is horizontal.

The Derivative as a Rate of Change

$$f'(a) = \text{rate of change of } f(x) \text{ at } x = a.$$

EXAMPLE

Let $S(x)$ represent the total sales (in thousands of dollars) for month x in the year 2005 at a certain department store. Represent each statement below by an equation involving S or S'.

(a) The sales at the end of January reached \$120,560 and were rising at the rate of \$1500 per month.

(b) At the end of March, the sales for this month dropped to \$80,000 and were falling by about \$200 a day (Use 1 month = 30 days).

The Derivative as a Rate of Change

SOLUTION

(a) Since the sales at the end of January (the first month, so $x = 1$) reached \$120,560 and $S(x)$ represents the amount of sales for a given month, we have: $S(1) = 120{,}560$. Further, since the rate of change of sales (rate of change means we will use the derivative of $S(x)$) for the month of January is a positive \$1500 per month, we have: $S'(1) = 1500$.

(b) At the end of March (the third month, so $x = 3$), the sales dropped to \$80,000. Therefore, sales for the month of March was \$80,000. That is: $S(3) = 80{,}000$. Additionally, since sales were dropping by \$200 per day during March, this means that the rate of change of the function $S(x)$ was (30 days) × (–200 dollars) = –6000 dollars per month. Therefore, we have: $S'(3) = -6000$.

Differentiating Various Independent Variables

EXAMPLE

Find the first derivative of $T = (1 + 2t)^2 + t^3$.

SOLUTION

$$\frac{dT}{dt} = \frac{d}{dt}\left[(1+2t)^2 + t^3\right]$$ We begin to differentiate.

$$\frac{dT}{dt} = \frac{d}{dt}(1+2t)^2 + \frac{d}{dt}t^3$$ Use the Sum Rule.

$$\frac{dT}{dt} = 2(1+2t)^1 \cdot \frac{d}{dt}(1+2t) + 3t^2$$ Use the General Power Rule.

$$\frac{dT}{dt} = 2(1+2t)^1 \cdot 2 + 3t^2$$ Finish differentiating.

$$\frac{dT}{dt} = 4(1+2t) + 3t^2$$ Simplify.

$$\frac{dT}{dt} = 4 + 8t + 3t^2$$ Simplify.

Second Derivatives

EXAMPLE

Find the first and second derivatives of $f(P) = (3P + 1)^5$.

SOLUTION

$f(P) = (3P+1)^5$ This is the given function.

$f'(P) = 15(3P+1)^4$ This is the first derivative.

$f''(P) = \frac{d}{dP}[f'(P)] = \frac{d}{dP}\left[15(3P+1)^4\right] = 180(3P+1)^3$ This is the second derivative.

Second Derivatives Evaluated at a Point

EXAMPLE

Compute the following: $\left.\frac{d^2}{dx^2}\left(3x^4+4x^2\right)\right|_{x=2}$

SOLUTION

$$\frac{dy}{dx}=\frac{d}{dx}\left(3x^4+4x^2\right)=12x^3+8x$$

Compute the first derivative.

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}\left(12x^3+8x\right)=36x^2+8$$

Compute the second derivative.

$$\left.\frac{d^2y}{dx^2}\right|_{x=2}=\left(36x^2+8\right)\Big|_{x=2}=36(2)^2+8=36\cdot 4+8=152$$

Evaluate the second derivative at $x = 2$.

Instantaneous Rate of Change

The derivative $f'(a)$ measures the rate of change of $f(x)$ at $x = a$.

EXAMPLE

Suppose that $f(x) = -6/x$. What is the (instantaneous) rate of change of $f(x)$ when $x = 1$?

SOLUTION

The rate of change of $f(x)$ at $x = 1$ is equal to $f'(1)$. We have

$$f'(x) = \frac{6}{x^2}$$

$$f'(1) = \frac{6}{1^2} = \frac{6}{1} = 6.$$

That is, the rate of change is 6 units per unit change in x.

Position, Velocity & Acceleration

If $s(t)$ denotes the position function of an object moving in a straight line, then the velocity $v(t)$ of the object at time t is given by

$$v(t) = s'(t).$$

$$a(t) = v'(t)$$

$s(t)$ is the position function, $v(t)$ is the velocity function, and $a(t)$ is the acceleration function.

Position, Velocity & Acceleration

EXAMPLE

A toy rocket fired straight up into the air has height $s(t) = 160t - 16t^2$ feet after t seconds.

(a) What is the rocket's initial velocity (when $t = 0$)?

(b) What is the velocity after 2 seconds?

(c) What is the acceleration when $t = 3$?

(d) At what time will the rocket hit the ground?

(e) At what velocity will the rocket be traveling just as it smashes into the ground?

SOLUTION

(a) To determine what the rocket's initial velocity is, we must first have a velocity function and then evaluate $t = 0$ in that function.

$s(t) = 160t - 16t^2$ This is the given position function.

$v(t) = s'(t) = 160 - 32t$ Differentiate to get the velocity function.

Position, Velocity & Acceleration

CONTINUED

$v(0) = 160 - 32(0) = 160 - 0 = 160$ Now replace t with 0 and evaluate.

Therefore, the initial velocity of the rocket is 160 feet per second.

(b) To determine the velocity after 2 seconds we evaluate $v(2)$.

$v(t) = 160 - 32t$ This is the velocity function.

$v(2) = 160 - 32(2) = 160 - 64 = 96$ Replace t with 2 and evaluate.

Therefore, the velocity of the rocket after 2 seconds is 96 feet per second.

(c) To determine the acceleration when $t = 3$, we must first find the acceleration function.

$v(t) = 160 - 32t$ This is the velocity function.

$a(t) = v'(t) = -32$ Differentiate to get the acceleration function.

Since the acceleration function is a constant function, the acceleration of the rocket is a constant -32 ft/s^2. Therefore, the acceleration when $t = 3$ is -32ft/s^2.

Position, Velocity & Acceleration

CONTINUED

(d) To determine at what time the rocket will hit the ground we must determine what we know about the position, velocity, or acceleration of the rocket when the rocket hits the ground. We know that at the time of impact, the position of the rocket will be 0 feet above the ground. Therefore, we will use the position function and replace $s(t)$ with 0.

$s(t) = 160t - 16t^2$	This is the given position function.
$0 = 160t - 16t^2$	Replace $s(t)$ with 0.
$0 = 10t - t^2$	Factor 16 out of both terms on the right and divide both sides by 16.
$0 = t(10 - t)$	Factor.
$t = 0 \quad t = 10$	Solve for t.

Therefore, the rocket will be 0 feet above the ground at times $t = 0$ and $t = 10$. $t = 0$ corresponds to when the rocket first began its flight, so that would not be the solution. So, the rocket hit the ground after 10 seconds, when $t - 10 = 0$.

Position, Velocity & Acceleration

CONTINUED

(e) To determine at what velocity the rocket will be traveling just as it smashes into the ground, we must use the velocity function. The question is, what do we use for t? From part (d), we know that the rocket will hit the ground at $t = 10$ seconds. Therefore, we will find $v(10)$.

$$v(t) = 160 - 32t$$ This is the velocity function.

$$v(10) = 160 - 32(10) = 160 - 320 = -160$$ Replace t with 10 and evaluate.

Therefore, when the rocket hits the ground, it will be have a velocity of -160 ft/s. That is, it will be traveling 160 ft/s in the downward direction.